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THE ANOMALY AND REGGEON FIELD THEORY IN QCD*

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The appearance of the U(1) anomaly in the interactions of reggeized gluons is described. Also discussed is the crucial role the anomaly can play in providing the non-perturbative properties necessary for a transition from gluon and quark reggeon diagrams to hadron reggeons and a reggeon field theory description of the pomeron.

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1. INTRODUCTION

In this talk I will report on a program[1,2], the goal of which is, to find a (candidate) regge region QCD S-Matrix starting from reggeized gluons and quarks (reggeons). We look for hadrons and the pomeron as bound-states of quark and gluon reggeons that appear in multi-regge amplitudes. The dynamics involved will be the infra-red divergences that occur in reggeon diagrams when both gluons and quarks are massless.

To obtain a unitary pomeron, we anticipate that Reggeon Field Theory (RFT), and in particular the critical pomeron[3], will be essential. RFT describes the pomeron as a regge pole plus multipomeron exchanges and interactions, in agreement with experiment, but not with perturbative QCD. The critical pomeron is the only known solution of s- and t-channel unitarity that gives a rising total cross-section and a connection with QCD could have important consequences. Firstly, since it allows physical cross-sections to have scaling behavior (up to logarithms), a rising total cross-section should correspond to the maximal applicability of asymptotic freedom to short distance processes. In addition, the factorization properties of the critical pomeron provide a "universal wee-parton distribution" that could carry the (vacuum) properties of confinement and chiral-symmetry breaking as part of an extension of the parton model beyond leading-twist perturbation theory.

Since our reggeon diagram starting point is essentially perturbative it might be expected that the "non-perturbative" properties of confinement and chiral-symmetry breaking will not appear at all in our formalism. In this talk, however, we will show that the U(1) anomaly appears in particular interactions of gluon reggeons[2]. The anomaly represents the potential ultra-violet/infra-red flow of chirality into and out of the theory and we will indicate how, when appropriately treated, it gives an infra-red divergence that can combine with the infra-red divergences of gluon reggeon diagrams to produce a transition to hadron and pomeron reggeon diagrams in which the desired non-perturbative properties appear (as properties of the spectrum).

2. REGGEON DIAGRAMS FROM SPONTANEOUSLY-BROKEN QCD

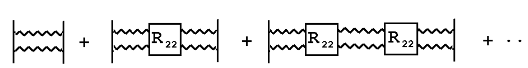
We begin by summarizing well-known results[4] from perturbative calculations in spontaneously-broken gauge theories. The regge limit of an elastic scattering amplitude, i.e. $s \to \infty$, t fixed, is studied in the complex angular momentum J-plane via the (Sommerfeld-Watson) representation

$$A(s,t) \; = \; \int \; dJ \; a(J,t) \; s^J \qquad (1)$$

When all gluons and quarks have a mass there are no infra-red divergences and the regge behavior is straightforward. Leading-log calculations show that both gluons and quarks "reggeize". In particular, the gluon becomes a regge pole with trajectory $J = 1 - \Delta(t)$. Non-leading logs are reproduced by "reggeon diagrams", which are $k_\perp$ integrals with gluon particle poles in addition to reggeon propagators. For example, the two gluon reggeon state appears in $a(J,t)$ as

$$\int \frac{d^2k_1}{k_1^2+M^2} \; \frac{d^2k_2}{k_2^2+M^2} \; \frac{\delta^2(Q-k_1-k_2)}{J-1+\Delta(k_1^2)+\Delta(k_2^2)} \qquad (2)$$

Reggeon unitarity[1,5,6] requires that a complete set of reggeon diagrams, involving all possible J-plane multi-reggeon states, appear in higher-orders. The well-known BFKL equation is a simple consequence of 2-reggeon unitarity, i.e. if we denote the 2-reggeon state by this equation sums the set of diagrams



obtained by iterating the $2-2$ reggeon interaction

$$\begin{aligned} R_{22} &= [(\underline{k}_1^2 + M^2)(\underline{k}_2^{2'} + M^2) \\ &+ (\underline{k}_2^2 + M^2)(\underline{k}_1^{2'} + M^2)]/[(\underline{k}_1 - \underline{k}_1')^2 + M^2] \\ &+ \; \cdots \end{aligned} \qquad (3)$$

When $M \to 0$, infra-red divergences exponentiate to zero all diagrams with non-zero t-channel color. Since the gluon poles remain, however, the

color zero amplitudes scale canonically ($\sim k_\perp^{-2}$) , i.e. there is no confinement! Our aim is to see the gluon poles disappear via an RFT phase-transition from quark and gluon to hadron and pomeron reggeon diagrams.

In general, reggeon unitarity requires[1,6] that quark and gluon reggeon diagrams describe not only elastic scattering but also all multi-regge limits of multiparticle amplitudes. Consequently we can study limits of high-order amplitudes, an example of which is illustrated in Fig. 1, where both hadrons and the pomeron could appear as (coupled) bound states of quark and gluon reggeons. Since we expect the anomaly to be involved and we consider it's infra-red manifestation, we look for such bound states when both the gluon mass M and the quark mass $m \to 0$. As we will see, both the U(1) anomaly and RFT phase-transition analysis are essential for understanding the infra-red divergence structure we find. To provide motivation and to describe features that we will be

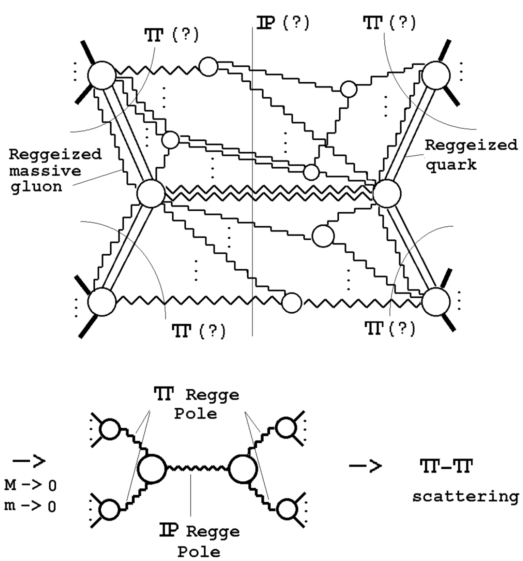


Fig. 1 An amplitude in which both pion and pomeron regge poles can appear.

looking for, we first introduce the RFT critical pomeron and an associated supercritical phase.

3. POMERON RFT

For a regge pole pomeron, RFT can be formulated directly from reggeon unitarity. The diagrams are essentially the same[1] as for reggeized gluons, but without the gluon poles!

3.1. The Critical Pomeron

This is a renormalization group fixed-point solution[3] of RFT in which $\Delta(0) = 1 - \alpha_{I\!P}(0) = 0$ and total cross-sections rise asymptotically

$$\sigma_T \underset{s\to\infty}{\sim} [\ln s]^\eta \qquad (4)$$

with $\eta = \frac{\epsilon}{12} + O(\epsilon^2)$, where $4-\epsilon$ is the transverse momentum dimension. Also, all other asymptotic predictions satisfy unitarity in both the t-channel and the s-channel.

3.2. The Supercritical Pomeron

To find the new phase that appears at the critical point, we consider the critical lagrangian

$$\mathcal{L} = \frac{1}{2}\bar{\phi}\overleftrightarrow{\frac{\partial}{\partial y}}\phi - \alpha_0'\nabla\bar{\phi}\nabla\phi - \Delta_0\bar{\phi}\phi - \frac{1}{2}ir_0\left[\bar{\phi}\phi^2 + \bar{\phi}^2\phi\right] \qquad (5)$$

where $\bar{\phi}$ (ϕ) creates (destroys) pomerons. The stationary point

$$\phi = \bar{\phi} = \frac{2i\Delta_0}{3r_0} \qquad (6)$$

gives a pomeron condensate that shifts the pomeron intercept ($\Delta_0 \to -\Delta_0/3$) and produces a solution[6] for $\Delta_0 < 0$ $(\alpha_{I\!P}(0) > 1)$. To determine the resulting graphical expansion the pomeron condensate has to be be interpreted as a "wee-parton" component of the scattering states[6]. (This solution was very controversial 20 years ago - although it was supported by Gribov !)

The condensate generates new classes of RFT diagrams, a simple example of which is shown in Fig. 2.

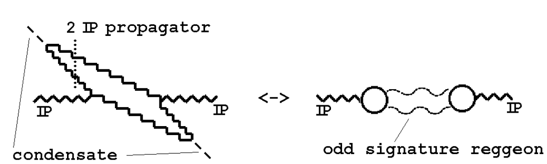


Fig. 2 A new RFT diagram generated by the pomeron condensate

As illustrated in this diagram, the two pomeron propagators produced by the condensate (i.e.

$[2\Delta_0 - 2\alpha_0' \; k_\perp^2 + \cdots]^{-1}$) give $k_\perp$ poles that have to be interpreted as particle poles, implying that there is a pomeron transition to a two vector reggeon state as shown. Reggeon states involving many vector particle poles similarly appear in higher-order diagrams.

In general, divergences in rapidity produced in the original graphical expansion because $\Delta_0 < 0$ are converted to vector particle divergences in $k_\perp$ in the supercritical expansion. Therefore, the supercritical phase is characterized by the "deconfinement of a vector particle on the pomeron trajectory".

3.3. The Supercritical Phase and Color Superconductivity

An immediate question is whether the gluon particle poles in QCD could disappear via the reverse of the deconfinement process just described. The appearance of a reggeized vector particle suggests that the supercritical phase could be realized in QCD via the breaking of the gauge symmetry from SU(3) to SU(2). In current terminology the supercritical pomeron would correspond to "color superconducting QCD". Identification of the vector mass with the RFT order parameter would then determine that the critical pomeron appears as the gauge symmetry is restored. Therefore, to look for the critical pomeron we begin by studying superconducting QCD.

The breaking of SU(3) gauge symmetry to SU(2) produces an (odd signature) SU(2) singlet, massive, reggeized gluon. An even signature pomeron would be produced if this massive reggeon appears in an SU(2) singlet reggeon condensate containing an odd number (≥ 3) of massless gluon reggeons. The corresponding SU(3) pomeron would contain an even number (≥ 4) of gluons but carry odd ("anomalous") color parity, in contrast to the even color parity (≥ 2 gluon) BFKL pomeron!

A three gluon condensate carries the quantum numbers of the winding-number current and so could be due to spectral flow of the Dirac sea produced by the anomaly. If we can understand the origin of a condensate of this kind then, provided the identification with the supercritical pomeron can be made, the disappearance of the condensate will give the critical pomeron we are looking for. For this purpose, we must determine how the anomaly appears in the regge-limit effective theory described by gluon and quark reggeon diagrams? (It is, of course, absent in the usual perturbation expansion of a vector theory.) The rest of this talk will be primarily devoted to this issue. The consequences (RFT for the pomeron, confinement and chiral symmetry breaking etc.) will be outlined only briefly.

4. THE ANOMALY IN TRIPLE-REGGE VERTICES

The simplest multi-regge limit in which the anomaly makes an appearance is the full triple-regge limit[7]. It is present in the "helicity-flip" part[2] of reggeized gluon interactions containing a single quark loop. (That bound-state couplings involve helicity-flip effects is directly related to the occurrence of chiral symmetry breaking.)

Consider the 3-3 scattering of quarks, each of which has a large light-cone momentum, but with the spacelike components of the momenta orthogonal, i.e.

$$\begin{aligned} P_1 \;&\to P_1^+ \;=\; (p_1, p_1, 0, 0)\;, \quad p_1 \to \infty \\ P_2 \;&\to P_2^+ \;=\; (p_2, 0, p_2, 0)\;, \quad p_2 \to \infty \\ P_3 \;&\to P_3^+ \;=\; (p_3, 0, 0, p_3)\;, \quad p_3 \to \infty \end{aligned} \qquad (7)$$

with the Q_i finite. We consider diagrams of the form illustrated in Fig. 3.

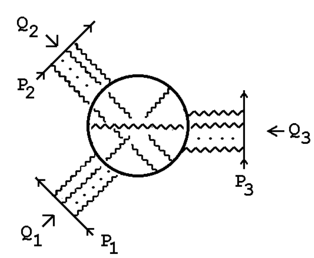


Fig. 3 Diagrams containing a single quark loop.

Using light-cone co-ordinates, the leading behaviour is obtained by putting quark lines on-shell via $k_{i\pm}$ integrations leaving the $k_\perp$-integrals of reggeon diagrams, as illustrated in Fig. 4.

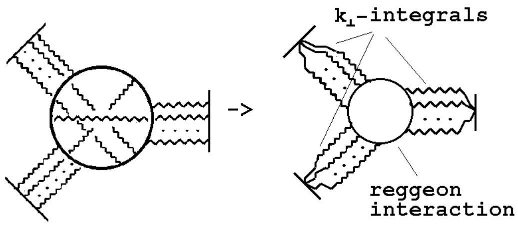


Fig. 4 The reduction to reggeon diagrams.

The resulting "triple-regge" reggeon interactions are quark triangle diagrams with both local (point-like) and non-local "effective vertices" containing γ-matrix products. In some some cases $\gamma_5\gamma$ couplings appear that, potentially, could generate the triangle anomaly.

A particular example, which we consider in more detail below, is the "maximally non-planar" diagram shown in Fig. 5. It is well-known that non-planar diagrams provide the essential structure of regge cut couplings. In a gauge theory other diagrams also contribute but, essentially, produce only subtraction effects that cancel large momentum divergences of the non-planar contributions. The anomaly could, however, be subject to such cancelations.

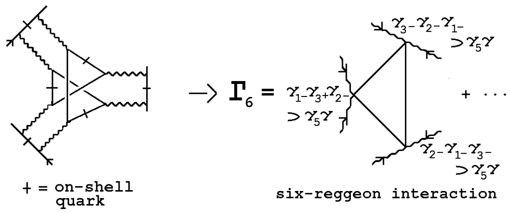


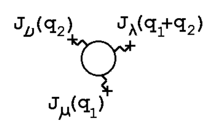
Fig. 5 The six-reggeon interaction obtained from a maximally non-planar diagram.

At lowest-order (i.e. two gluons in each Q_i channel) there are $\sim$ 100 diagrams that potentially could give contributions. Many obviously do not contain the anomaly while others, in particular the maximally non-planar diagrams, give several contributions. To systematically evaluate all contributions and also to discuss cancelations a triple-regge asymptotic dispersion relation formalism, in which multiple discontinuities are initially calculated rather than amplitudes, can be used[2]. A crucial feature of this formalism is the contribution, to the dispersion relation, of unphysical triple discontinuities that contain chirality transitions. The absence of the anomaly in simpler multi-regge limits can be understood as due to the absence of such contributions in the corresponding asymptotic dispersion relation.

We will not describe the dispersion relation formalism in this talk. Full details can be found in [2]. Instead we will study a maximally non-planar diagram directly. We will see, however, how the anomaly is associated with an unphysical multiple discontinuity. First we describe the infra-red properties of the anomaly that we will look for.

5. THE ANOMALY AS AN INFRA-RED DIVERGENCE

To avoid the ultra-violet subtleties of reggeon interactions, and to see a connection with spectral flow, we will look for the anomaly in the infra-red region[8]. For a massless quark axial-vector current the anomaly divergence equation for the three-current vertex gives



$$\sim \ \epsilon_{\nu\lambda\alpha\beta}\frac{q_{1\mu}q_2^{\alpha}(q_1+q_2)^{\beta}}{q^2}+\cdots \qquad (8)$$

in the limit $q_1^2 \sim q_2^2 \sim (q_1+q_2)^2 \sim q^2 \ \rightarrow 0$. (For the U(1) current we ignore non-perturbative contributions. We are looking for a "perturbative" effect!)

If $q_{1+} \not\rightarrow 0$ and q_2 is spacelike with $q_2 \perp q_{1+}$

$$\epsilon_{\nu\lambda\alpha\beta}\ \frac{q_{1\mu}q_2^{\alpha}q_1^{\beta}}{q^2} \ \sim\ \frac{{q_{1+}}^2}{q} \ \sim\ \frac{1}{q} \qquad (9)$$

(In the absence of a Lorentz-covariant separation of kinematic factors, this linear divergence can be used to characterize the anomaly.)

As illustrated in Fig. 6, the divergence is produced by a zero momentum chirality transition (corresponding to spectral flow) combined with a light-like momentum q_{1+} flowing through the other two propagators in the loop.

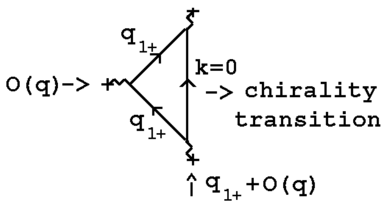


Fig. 6 The momentum configuration producing the anomaly.

Because the chirality transition in the quark loop is essential, the anomaly infra-red divergence can not be canceled by gluon diagrams.

6. A MAXIMALLY NON-PLANAR DIAGRAM

In this Section we give an abbreviated version of the calculation, presented in detail in [2], of the triple-regge limit of the maximally non-planar diagram shown in Fig. 5. We can redraw the diagram and label momenta as in Fig. 7.

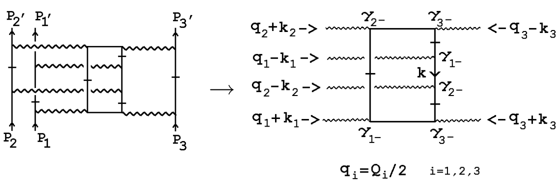


Fig. 7 Momenta in a maximally non-planar miagram.

For the k_1 and k_2 integrations we use the special light-cone co-ordinates ("generalized Sudakov variables")

$$k_i = k_{i2-}\ \underline{n}_{1+} + k_{i1-}\ \underline{n}_{2+} + \underline{\tilde{k}}_{i12} \qquad i = 1,2$$

where $\underline{n}_{1+} = (1,1,0,0)$ and $\underline{n}_{2+} = (1,0,1,0)$, together with the conventional light-cone co-ordinates $(k_{3+}, k_{3-}, \underline{\tilde{k}}_{3\perp})$ for the k_3 integration. The k_{11-}, k_{22-}, k_{3-} integrations are straightforward. The six options for using the remaining longitudinal k_i integrations to put hatched lines on-shell are shown in Fig. 8.

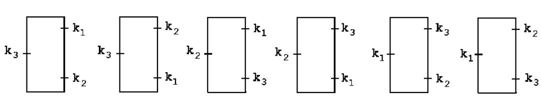


Fig. 8 Options for the longitudinal integrations.

We look for the anomaly to be generated by local $\gamma_5\gamma$ couplings. A "local" coupling is obtained from the component of a quark numerator with the same momentum factor that scales the corresponding integrated longitudinal momentum. In the chosen co-ordinates, only the first option in Fig. 8 gives couplings, all three of which have the required components. In this case we obtain

$$\begin{aligned}
&\int dk_{12-}\ \delta\Big((k_1+k-q_1)^2 - m^2\Big) \\
&\quad \gamma_{3-}\ \Big((k_1+k-q_1)\cdot\gamma + m\Big)\ \gamma_{1-} \\
&= \int dk_{12-}\ \delta\Big(k_{1-}\ k_{12-} + \cdots\Big) \\
&\quad \gamma_{3-}\ \Big(k_{1-}\gamma_{2-} + \cdots\Big)\ \gamma_{1-} \\
&= \gamma_{3-}\gamma_{2-}\gamma_{1-} \quad + \quad \cdots
\end{aligned} \tag{10}$$

$$\begin{aligned}
&\int dk_{21-}\delta\Big((k_2-k-q_2)^2 - m^2\Big) \\
&\quad \gamma_{2-}\ \Big((k_2-k-q_2)\cdot\gamma + m\Big)\ \gamma_{3-} \\
&= \int dk_{21-}\ \delta\Big(k_{2-}k_{21-} + \cdots\Big) \\
&\quad \gamma_{2-}\ \Big(k_{2-}\gamma_{1-} + \cdots\Big)\ \gamma_{3-} \\
&= \gamma_{2-}\gamma_{1-}\gamma_{3-} \quad + \quad \cdots
\end{aligned} \tag{11}$$

$$\begin{aligned}
&\int dk_{33+}\ \delta\Big((k_3+k+k_1-k_2)^2 - m^2\Big) \\
&\quad \gamma_{1-}\ \Big((k_3+k+k_1-k_2)\cdot\gamma + m\Big)\ \gamma_{2-} \\
&= \int dk_{33+}\ \delta\Big((k_{3-}+k_{13-}-k_{23-})k_{33+} + \cdots\Big) \\
&\quad \gamma_{1-}\ \Big((k_{3-}+k_{13-}-k_{23-})\gamma_{3+} + \cdots\Big)\ \gamma_{2-} \\
&= \gamma_{1-}\gamma_{3+}\gamma_{2-} \quad + \quad \cdots
\end{aligned} \tag{12}$$

Writing

$$\begin{aligned}
\hat{\gamma}_{31} &= \gamma_{3-}\gamma_{2-}\gamma_{1-} = \gamma^{-,+,-} - i\gamma_5\gamma^{-,-,-} \\
\hat{\gamma}_{23} &= \gamma_{2-}\gamma_{1-}\gamma_{3-} = \gamma^{+,-,-} - i\gamma_5\gamma^{-,-,-} \\
\hat{\gamma}_{12} &= \gamma_{1-}\gamma_{3+}\gamma_{2-} = \gamma^{-,-,-} + i\gamma_5\gamma^{-,-,+}
\end{aligned} \tag{13}$$

where
$$\begin{aligned}\gamma^{\pm,\pm,\pm} &= \gamma \cdot n^{\pm,\pm,\pm} \\ n^{\pm,\pm,\pm} &= (1, \pm 1, \pm 1, \pm 1)\end{aligned} \tag{14}$$
the part of the asymptotic amplitude with local couplings is
$$\begin{aligned}& g^{12}\frac{p_{11+}p_{22+}p_{3+}}{m^3}\int \frac{d^2\underline{k}_{112+}}{(q_1+\underline{k}_{112+})^2(q_1-\underline{k}_{112+})^2} \\ & \int \frac{d^2\underline{k}_{212}}{(q_2+\underline{k}_{212+})^2(q_2-\underline{k}_{212+})^2} \\ & \int \frac{d^2\underline{k}_{33\perp}}{(q_3+\underline{k}_{33\perp})^2(q_3-\underline{k}_{33\perp})^2} \\ & \int d^4k \; \frac{Tr\{\hat{\gamma}_{12}(\not k+\not k_1+\not q_2+\not k_3+m)\hat{\gamma}_{31}}{([k+k_1+q_2+k_3]^2-m^2)} \\ & \frac{(\not k+m)\hat{\gamma}_{23}(\not k-\not k_2+\not q_1+\not k_3+m)\}}{(k^2-m^2)([k-k_2+q_1+k_3]^2-m^2)}\end{aligned} \tag{15}$$
For simplicity, we have set the gluon mass to zero. This means that the transverse momentum integrals are actually infra-red divergent. However, since our next step is to remove both these integrals and the gluon propagators as the lowest-order contribution of three two-reggeon states, these divergences play no role in the present discussion. The general structure of gluon transverse momentum divergences will, of course, play a vital role in the full analysis of reggeon diagrams that we outline in the final Section.

Removing the transverse integrals and the gluon propagators, as illustrated in Fig. 5, we obtain a triangle diagram six-reggeon vertex Γ_6. The component with three γ_5 couplings is the $(m=0)$ triangle diagram illustrated in Fig. 9

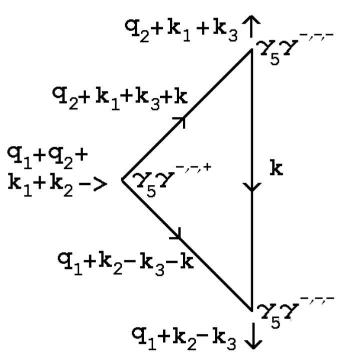


Fig. 9 The triangle diagram giving the anomaly in Γ_6.

while the terms with a single γ_5 coupling have the wrong kinematic structure[2]. Therefore, we can write
$$\begin{aligned}\Gamma_6(q_1,q_2,q_3,\tilde{\underline{k}}_1,\tilde{\underline{k}}_2,\underline{k}_{3\perp},0) \quad &= \\ \int d^4k \frac{Tr\{\gamma_5\gamma^{-,-,+}(\not k+\not k_1+\not q_2+\not k_3)\gamma_5\gamma^{-,-,-}}{(k+k_1+q_2+k_3)^2 \; k^2} & \\ \frac{\not k\gamma_5\gamma^{-,-,-}(\not k-\not k_2+\not q_1+\not k_3)}{(k-k_2+q_1+k_3)^2} \;+\; \cdots &\end{aligned} \tag{16}$$
where the remainder of the vertex does not contain the anomaly.

The anomaly divergence appears in the limit
$$\begin{aligned}&(q_2+k_1+k_3)^2 \sim (q_1+q_2+k_1+k_2)^2 \\ &\sim (q_1+k_2-k_3)^2 \sim q^2 \to 0\end{aligned} \tag{17}$$
with $q_2+k_1+k_3$ $(=k_1-q_1+k_3-q_3)$ having a finite light-like component. Mass-shell constraints resulting from the longitudinal integrations, that must be satisfied, are
$$\begin{aligned}(k-q_1+k_1)^2 &= (k+q_2-k_2)^2 \\ &= (k+k_1-k_2+k_3)^2 = 0\end{aligned} \tag{18}$$
All constraints are satisfied, with $k=0$, if $q_1^2 = q_2^2 = k_1^2 = k_2^2$ and
$$\begin{aligned}(q_1-k_1) &\to -2l(1,1,0,0) \\ (q_2-k_2) &\to \;\;\, 2l(1,0,1,0) \\ q_3 &\to \;\;\;\; l(0,1,-1,0) \\ k_3 &\to -l(0,1-2cos\theta_{lc},1-2sin\theta_{lc},0)\end{aligned} \tag{19}$$
In the limit $q \to 0$, only the light-like vector $k_{lc} = 2l(1,cos\theta_{lc},sin\theta_{lc},0)$ flows through the triangle and the anomaly gives
$$\Gamma_6 \quad \sim \quad \frac{(1-cos\theta_{lc}-sin\theta_{lc})^2 \; l^2}{q} \tag{20}$$
It is important that although the mass-shell constraints are satisfied in this momentum configuration, they do not correspond to physical region discontinuities for the scattering process of Fig. 7. Instead, as noted above, and discussed in detail in [2], the corresponding multiple discontinuity is unphysical.

7. PROPERTIES OF THE ANOMALY

7.1. The Space-time Picture

The momentum configuration described in the previous Section corresponds to the physical scattering process shown in Fig. 10, which we refer to as the "basic anomaly process".

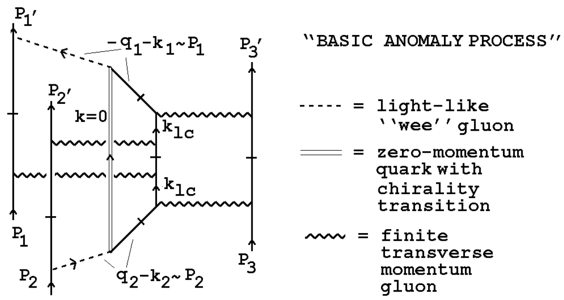


Fig. 10 The physical scattering giving the anomaly.

With the time direction up the page we obtain the following space-time picture. An incoming quark emits a "wee gluon" that produces a zero-momentum quark together with a light-like antiquark. The zero-momentum quark undergoes a chirality transition while the antiquark's lightlike momentum is rotated to k_{lc} by a t_3-channel gluon. The antiquark then forward scatters off a gluon from the t_1 channel before another t_3-channel gluon again rotates the antiquark light-like momentum so that it and the zero-momentum quark can annihilate into an outgoing wee gluon with spacelike momentum orthogonal to the initial incoming wee gluon.

We emphasize that the scattering process of Fig. 10, with the chirality transition, enters the physical region only asymptotically and then only when the quark mass is zero.

7.2. Reggeon Ward Identities

We anticipate that gauge invariance will be manifest via reggeon Ward identity cancelations[1] that could involve the anomaly. For example, the diagram of Fig. 11 cancels the anomaly in the diagram of Fig. 10 when the "wee gluons" are in a color zero state in the t_3-channel.

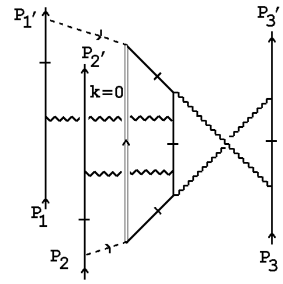


Fig. 11 A diagram related to Fig. 10 by a reggeon Ward identity

In a color octet state the Ward identity involves a gluon interaction which can not contain a chirality transition. As a result, there is no cancelation and the reggeon Ward identity is violated. A non-abelian gauge symmetry is crucial, therefore, for the non-cancelation of the anomaly.

7.3. Parity Cancelations

If an alternative set of quark lines is placed on-shell in the original maximally non-planar diagram of Fig. 5, a further anomaly contribution is obtained. As shown in Fig. 12, a parity transformation interchanging P_1 and P_2 relates the two contributions and hence the anomaly has the opposite sign.

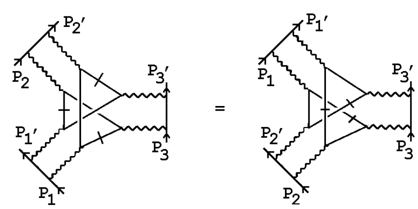


Fig. 12 Another anomaly contribution with P_1 and P_2 interchanged.

This does not produce a cancelation in the reggeon interaction we have extracted, but does in the complete diagram after the transverse momentum integrations are performed.

7.4. When is there no cancelation?

Because of the discontinuity structure of the amplitudes in which it is contained[2], the anomaly vertex conserves signature (producing, we anticipate, an even signature pomeron in hadron amplitudes). Because of it's parity properties the anomaly couples only anomalous color

parity (i.e. not equal to the signature) gluon reggeon combinations. Such exchanges do not couple to elementary quark or gluon scattering states and so, in effect, the anomaly cancels to all orders for such states. (Note, however, that the cancelation occurs after transverse momentum integrations have been performed, not in the reggeon interaction.)

With clusters (potentially forming bound states) in the initial or final states there can be sufficient structure in the couplings to the exchanged reggeons that the parity properties of the anomaly do not produce a cancelation. The anomaly infra-red divergence will be suppressed by the reggeon Ward identity zeroes of such couplings, but there will also be ultra-violet effects which we do not expect to be suppressed. Therefore we anticipate that, in amplitudes that contain the anomaly (there must be an even number of anomaly vertices for color parity to be conserved), ultra-violet effects will produce a power violation of unitarity bounds by reggeon exchanges. Indeed, we suspect that unitarity violation associated with anomaly interactions is a core problem for the existence of a bound state S-Matrix in a general gauge theory.

8. THE PROPOSED QCD SOLUTION

We are currently constructing a solution for QCD based on the properties of the anomaly discussed above. We can briefly describe the essential features that are emerging, as follows.

Using a generalization of the procedure outlined in Section 4, a complete set of multi-reggeon scattering amplitudes can be extracted from high-order particle amplitudes. In superconducting QCD, with SU(3) color broken to SU(2), all reggeon states with non-zero SU(2) color have infra-red divergences that exponentiate amplitudes to zero. If we consider initial scattering reggeon states, with anomalous color parity, that contain massive gluon reggeons (or quark reggeons) in a reggeon condensate, we obtain a sub-set of color zero amplitudes that has an overall logarithmic infra-red divergence. The divergence occurs when all triple-regge interactions that could contain the anomaly, do so. If the divergence is factored off the residue is a set of "physical reggeon amplitudes" in which the condensate appears also in all intermediate and final reggeon states (a completeness property for states containing the condensate). Ultra-violet effects of the anomaly are not present in the physical amplitudes.

The reggeon interactions remaining after the anomaly divergence is factored off have the general form shown in Fig. 13.

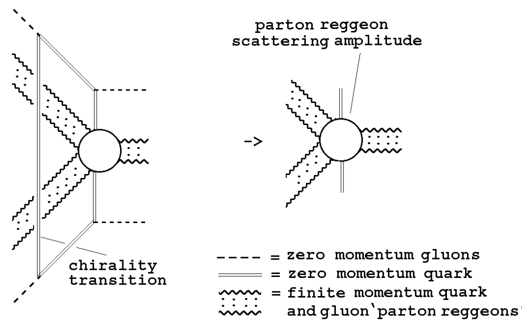


Fig. 13 The "parton reggeon interaction".

Comparing with the basic anomaly process of Fig. 10, we note that the complete quark loop containing the chirality transition carries zero momentum and that the forward scattering part of the interaction has been replaced by a tranverse momentum conserving, "parton reggeon interaction". The parton scattering lies in the broken part of SU(3), while the background anomaly interaction of zero momentum gluons lies in the unbroken part. These gluons carry the quantum numbers of the SU(2) winding number current in each t-channel. There is confinement of SU(2) gluons in that the "parton reggeons" do not include massless gluons - such states are amongst those exponentiated to zero.

The pomeron is a massive reggeon in the reggeon condensate and carries odd color parity as anticipated. The BFKL pomeron does not appear. The lowest-order triple-pomeron interaction is given by diagrams of the form shown in Fig. 14.

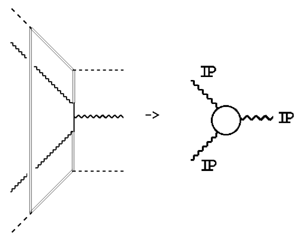


Fig. 14 A diagram contributing to the triple-pomeron interaction.

The reggeon condensate appears to have all the right properties to be identified with the pomeron condensate of super-critical RFT.

There are massless quark states that are the Goldstone bosons corresponding to chiral symmetry breaking. Since the unbroken gauge symmetry is SU(2), these include[9] both pions and "nucleons". The chiral symmetry breaking involves an additional chirality violation that is, in effect, an S-Matrix analogue of the appearance of a chiral condensate. Additional chirality transitions appear in $\pi-\pi$ scattering, within $k_\perp$ integrals as illustrated in Fig. 15, to compensate for the helicity-flip of the anomaly interaction. Although we will not discuss higher-order diagrams here, we hope to establish a complete correspondence between the reggeon diagrams describing $\pi-\pi$ scattering in superconducting QCD and the supercritical expansion described in Section 3.

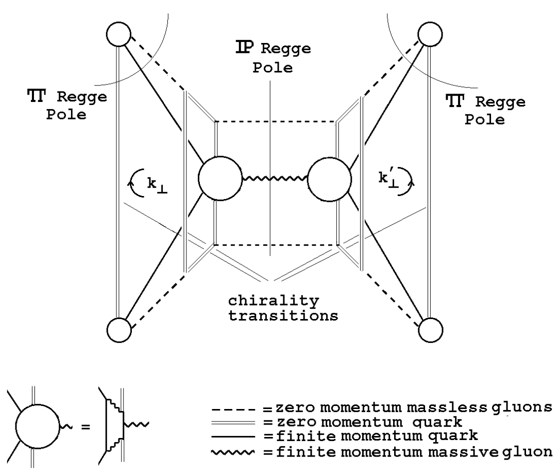


Fig. 15 $\pi-\pi$ scattering - via additional chirality transitions.

As SU(2) gauge symmetry is restored to SU(3) the condensate effects should disappear and leave behind critical pomeron behavior, as described by RFT. The decoupling of the odd signature reggeon will then complete the confinement of SU(3) gluons. The zero momentum quark and gluon interactions of Figs. 13 - 15 should be replaced by the interactions of a universal wee-parton distribution surrounding a "hard reggeon parton interaction". Thus providing the anticipated extension of the parton model.

The critical behavior will occur with no $k_\perp$ cut-off only when the gauge symmetry breaking from SU(3) to SU(2) does not destroy the asymptotic freedom of the theory. This requires extra quarks beyond those observed experimentally. In fact, the additional quarks required could be a color sextet quark sector that is responsible for electroweak symmetry breaking in the Standard Model. However, we will not enlarge on this possibility here.

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QUESTIONS

M. LOEWE (P. Universidad Cato'lica, Chile)

Question - Many years ago Bartels proposed a picture according to which the wee partons obey a diffusion equation. What is the relation of this picture and your triple pomeron vertex, including the anomaly?

Answer - Bartels' picture applies to the BFKL pomeron and the diffusion is in transverse momentum. Although I discussed the anomaly within a six-reggeon vertex that could couple three BFKL pomerons, it cancels in this context. It does appear in my triple-pomeron vertex, i.e. when the pomeron contains a minimum of four gluons and carries odd color parity. My pomeron is a regge pole and therefore has the corresponding diffusion picture in impact parameter space.

Question - Is there any simple reason why the maximal non-planar diagram is the relevant one?

Answer - It is well-known that non-planar diagrams have the double spectral function property necessary to produce regge cut couplings in a non-gauge theory. For the same reason non-planar diagrams provide the essential structure of fermion loop contributions to the (regge limit) interactions of photons in QED or gluons in QCD. The planar diagrams, in effect, simply regularize divergences appearing in the non-planar contributions. The role of the maximally non-planar diagram with respect to the anomaly is, in part, an extension of this situation. However, the anomaly also has a kinematic structure that is intrinsically four-dimensional and (essentially) the complexity of the maximally non-planar diagram is needed to generate this structure.

H. FRITZSCH (Universität München, Germany)

Question - There is also the non-perturbative contribution of instantons to the anomaly. How would instanton interactions affect your discussion?

Answer - The aim of my procedure is to discover and regularize the anomaly to produce an S-Matrix description of the pomeron and hadron reggeons that is unitary in the (multi-)regge region. I should then have the complete (multi-regge region) answer, including any contribution made by instanton interactions. To explicitly discuss instanton interactions it is necessary to start with the euclidean path-integral formulation of the theory, continue to Minkowski space, and then (if possible) extract the regge region S-Matrix. It may be, and indeed I expect, that there is a match only in the special circumstance that we consider SU(3) gauge theory with a particular fermion sector.